Rain Effects at Low Reynolds Number

J. F. Marchman III,* Edward A. Robertson,† and Howard T. Emsley† Virginia Polytechnic Institute and State University, Blacksburg, Virginia

Wind-tunnel tests were conducted to determine the influence of rain on the aerodynamic performance of the Wortmann FX63-137 wing at Reynolds numbers 100,000-300,000. Models with aspect ratios of 6 and 4 were tested with normal as well as waxed and soaped surfaces to evaluate wettability effects. Test results showed an improvement of performance at the lowest Reynolds number but general deterioration at higher speeds. The primary effect noted was a decrease in lift at higher angles of attack. A favorable result was the elimination of stall hysteresis, which characterizes the Wortmann's normal performance at low Reynolds numbers. At lower Reynolds numbers, rain appeared to lower drag whereas, at higher speeds, drag generally increased.

Introduction

In the past few years, interest has increased in two important areas of aerodynamic research. Growing public concern about weather-related aircraft accidents has spurred interest in determining the effects of the natural environment—and, in particular, of vertical gusts, or wind shear, and rain—on the aerodynamic performance of wings and aircraft. At the same time, increased activity in ultralight aircraft and remotely piloted vehicles (RPV's) has renewed interest in low Reynolds number aerodynamics. Since low Reynolds number flight is very sensitive to disturbances from environmental effects, such as rain or wind shear, an interesting focal point of both concerns is research into the effects of rain on a low Reynolds number airfoil.

Recent studies by Haines and Luers, ¹ Hansman and Barsotti, ² and Dunham et al. ³ have shown that rain causes significant reduction in maximum lift coefficient and generally alters stall characteristics. These results appear to be essentially due to the introduction of an effective surface roughness by the rain. This roughness effect has been shown in Ref. 2 to be a function of surface "wettability."

The primary result of added surface roughness is to increase the likelihood of laminar-turbulent transition. This subject has received considerable attention in the past. In recent years, however, the advent of airfoils designed for laminar flow and low Reynolds number operation has made the subject of roughness effects more important. Aircraft accidents involving vehicles with laminar flow design wings and canards have been associated with dramatic alteration of the wing or canard aerodynamics due to boundary-layer tripping caused by surface roughness or rain.

Wings designed to operate at low Reynolds numbers (below 500,000) may be especially susceptible to rain-induced aerodynamic deterioration. Studies by Marchman, Sumantran, and Schaefer⁴ have shown that an airfoil designed for low Reynolds number operation may have a very large stall hysteresis behavior and that this behavior may be altered by acoustic or flow turbulence disturbances. It is likely that rain would also lead to large changes in the stall behavior of a wing at low Reynolds number. The low Reynolds number stall hysteresis phenomenon is illustrated in Fig. 1 for a Wortmann

FX63-137 wing. After stall, the wing angle of attack must be significantly reduced in order to obtain flow reattachment.

It is conceivable that rain conditions may alter the performance of the Wortmann airfoil in several ways. The presence of water droplets in the flow affects airfoil performance primarily through a surface roughening effect as the fluid beads on the wing. The effects of momentum transfer from water droplets striking the wing and the increased weight of the wing due to the water film are considered less significant than the roughness effect for rainfall rates less than 500 mm/h.² According to Haines and Luers, rainfall rates >500 mm/h (calculated for a 1-min interval) are highly uncommon although rates as high as 1828.8 mm/h have been recorded. Typical short-duration rainfall rates are less than 100 mm/h. A recent report by Alan Bilanin⁵ supports the insignificance of raindrop momentum for reasonable rainfall rates, but it indicates that the cloud of minute droplets produced by rain impact may have a strong effect on the boundary layer. The cloud of droplets known as the "ejecta fog" drains energy from the boundary layer as the droplets are reaccelerated to local flow velocity. The loss of energy contributes to early flow separation and a resulting loss of lift.

According to Ref. 2, the size of the surface beads, and thus the magnitude of surface roughness, is directly related to the wettability of the surface material. A nonwettable (waxed) surface should exhibit a greater performance loss than a semiwettable (untreated) surface owing to the thicker beads of water. Similarly, the untreated surface should degrade performance more than a wettable (glycerin-soaped) surface under rain conditions. A uniformly wettable surface is difficult to prepare, however, due to irregularities in the soap coating. The soaped surface may actually produce a rougher water film and a greater performance loss than an untreated surface.

Hansman and Barsotti² performed experiments to simulate the water droplet roughness effect by artificially tripping the boundary layer. If the point of laminar-to-turbulent transition is moved toward the leading edge to simulate decreased wettability, variations in drag data can be accurately modeled when corrected for drop momentum transfer. The loss of lift and the decrease in the effective angle of attack, however, cannot be re-created by artificially inducing transition on a dry wing.

Hastings and Manuel⁶ found that the reduction in the effective angle of attack is a camber effect due to the uneven distribution of the water film. Their experiments showed that the continuous water film on the upper surface is considerably thinner than the film on the lower surface. Additionally, the point of maximum film thickness occurs farther aft on the lower surface.

Parameters of importance in water spray experiments are the Weber number, the liquid water content (LWC), and the

Presented as Paper 87-0258 at the AIAA 25th Aerospace Sciences Meeting, Reno, NV, Jan. 12-15, 1987; received Jan. 22, 1987; revision received April 5, 1987. Copyright © American Institute of Aeronautics and Astronautics, Inc., 1987. All rights reserved.

^{*}Professor of Aerospace & Ocean Engineering. Associate Fellow AIAA.

^{*†}Undergraduate Research Assistant. Student Member AIAA.

equivalent rainfall rate (RRe).² The Weber number relates dynamic pressure forces to surface tension forces and can be used to characterize zones of water flow on the wing surface. In the forward zone, shear forces due to the airflow result in a fairly uniform film of water. In the rear zone, particularly after flow separation has occurred, surface tension forces are dominant. Hastings and Manuel⁶ state that adverse changes in lift and/or drag are more likely to result from surface water on the top, rear of the wing, than from a continuous film on the front of the wing. The effects of beading in the aft zone are considered to be increased friction drag and earlier turbulent flow separation in the adverse pressure gradient. Because the Weber number depends on surface tension, the surface wettability of a wing has a direct influence on its performance under rain conditions.

The LWC is a parameter that relates the mass of water crossing a vertical plane at the wing to the volume of air flowing past the wing.³ Because water spray experiments do not generally simulate the drop size distribution present in natural rain, the LWC is preferred over a rainfall rate specification to denote the intensity of rain at the wing. However, an equivalent rainfall rate is often included which indicates what the natural rainfall rate would be for the given LWC. The equivalent rainfall rate is a generalization based on the integrated product of the LWC and the drop velocity.

Bilanin notes in Ref. 5 that the application of experimental rain effect data to various scale wings involves a complex interaction of aerodynamic parameters. Reducing the dimensional rain variables (neglecting compressibility) by the Buckingham pi theorem yields nine nondimensional groups. These groups involve the Reynolds number of air and water, the Weber number squared, surface tension and inertial forces, and rain geometry. Scaling of these parameters without the distortion of one or more nondimensional groups is prevented by velocity considerations. Thus, the application of subscale results to full-scale designs is limited until accurate scaling laws are devised. It is calculated by Bilanin⁵ that parameter imbalances in one-quarter scale tests may result in water film thicknesses several times too large. Reference 1 also notes that because the ratio of roughness element height to wing chord determines the decrease in the maximum lift coefficient, it is probable that full-scale performance losses will be much smaller than indicated by present subscale experiments.

Experimental Procedure

Tests were conducted in the Virginia Tech Stability Wind Tunnel, a closed-circuit facility with a $6 \times 6 \times 28$ ft test section. Seven sets of antiturbulence screens and an air exchange tower effectively minimized turbulence and acoustic influences in the testing environment. All tests used wings employing the Wortmann FX63-137 airfoil section (Fig. 2) mounted on a six-component, strain-gage strut in the wind tunnel. One test wing was constructed of styrofoam and balsa wood with a C-shaped steel spar for rigidity and strength. This wing had a Wortmann FX63-137 section with a 5-in. chord and an aspect ratio of 6, with no sweep and no taper. A second model used the same section and chord but was constructed of aluminum and had an aspect ratio of 4. The angle of attack of the model was manually controlled on command via a remote system using an electronic inclinometer. The actual angle sensed by the inclinometer was read by the data acquisition system. The data acquisition package consisted of a sixcomponent strain-gage balance, a Hewlett-Packard 3052A Automatic Data Acquisition System, and a Hewlett-Packard 9836 computer with full data processing and plotting capabilities. Using software developed by A. A. Abtahi and V. Sumantran of Virginia Tech, 25 readings were averaged for each data point. The data were automatically corrected for tares and reduced to coefficient form.

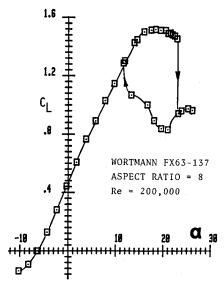


Fig. 1 Low Re stall hysteresis behavior.



Fig. 2 Wortmann FX63-137 section.

To provide the rain conditions, a streamlined tube (2-in. chord, 7/8-in. maximum thickness) incorporating seven ordinary garden sprayer nozzles was mounted horizontally 13 ft upstream of the model. Initially, the spray tube was placed in the same horizontal plane as the wing. Since an expected effect of rain was a reduction in the stall hysteresis effect normally seen at low Reynolds numbers, it was important to evaluate the effect of the spray system itself on tunnel turbulence and, hence, on stall hysteresis. Tunnel turbulence has been shown by Marchman et al.4 to be a primary contributor to hysteresis reduction. Hot-wire anemometer tests with this configuration indicated an apparently acceptable freestream turbulence of 0.06-0.07%. But subsequent tests with the wing models revealed that the stall hysteresis loop was destroyed by the presence of the spray tube, indicating that although overall turbulence levels were still low, the turbulence spectrum was such that boundary-layer instabilities were apparently sufficiently excited to greatly alter the character of the boundary layer. Moving the tube 18 in. above the plane of the wing restored the stall hysteresis loop while still providing a uniform spray over the wing itself.

Dry tests were first performed at Reynolds numbers of 100,000, 200,000, 300,000 to provide control data with the spray system in the tunnel. These data compared favorably with results from previous experiments at Virginia Tech^{4,7,8} showing only the changes one would expect from a very slight turbulence increase. The prestall lift and drag data were not noticeably altered from previous tests without the rain system in place, and there was only a very slight reduction in the extent of stall hysteresis from previous tests.

The seven nozzles on the spray tube were set for a large, uniform spray. Because the airflow constricted the spray area, the wide initial setting was necessary so that the nozzles would not have to be adjusted at high wind-tunnel velocities. Figure 3 shows details of the spray nozzle system, along with a view of the system at work in the wind tunnel. Although the droplet size distribution in the spray was not evaluated in these tests, its effects should have been minimized by this procedure. The maximum water flow rate through the ½-in. valve was 3.4 gal/min. The valve was kept fully open for all rain ex-

periments to avoid surging and to maintain a uniform spray. Unfortunately, because the LWC is based on the spray area and the freestream velocity, a constant LWC could not be maintained since the spray area was reduced as freestream speed increased.

The first rain tests were performed on the untreated wing, again for Reynolds numbers of 100,000, 200,000, and 300,000. The paint finish of the wing approximated the semiwettable surface of standard aircraft. A Reynolds number of 200,000 was chosen for the wettability experiments because of the low scatter in the data at that speed. The waxed surface (prepared with a liquid car wax) was much less wettable than the untreated surface and for the purposes of this experiment was considered nonwettable. Both the untreated and waxed surfaces were tested from an angle of -14 deg through stall in 2-deg increments.

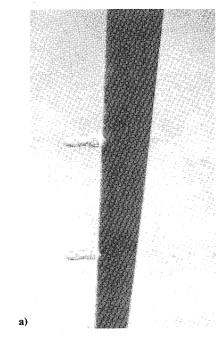
Attempts were made to test with a fully wettable surface by using a glycerin-based soap to coat the wing. Unfortunately, the soap very quickly washed away. Tests were conducted at a few isolated angles of attack on the aspect-ratio-6 wing; however, the inability to maintain a wettable surface for a reasonable period of time as the angle of attack was varied prevented any valid assessment of the effect on stall hysteresis of rain impacting a wettable surface. More extensive tests were conducted on the aspect-ratio-4 model.

Test Results

Initial testing involved the aspect-ratio-6 wing. The first series of tests evaluated the effect of rain on the wing with a normal, unwaxed surface. Testing was done at Reynolds numbers of 100,000, 200,000, and 300,000. At each Reynolds number, tests were conducted through a range of angle of attack from -14 deg through stall (or as high as 30 deg). All tests were run with the spray system in the tunnel; thus, any effects of that system on the flow were present in all tests.

Figure 4 shows the results of the tests at a Reynolds number of 100,000, and the data are seen to be quite different for the two cases. Some explanation of the "dry" wing data is in order, especially for those unfamiliar with low Reynolds number aerodynamic behavior. The "dry" wing results show a relatively linear C_L increase from -14 through about 6 deg with a slight dip in the curve around 0-deg angle of attack. In this range of α , a separation bubble (laminar separation followed by turbulent reattachment) exists on the wing's lower surface at the lower angles of attack. As angle of attack increases, a large laminar bubble over the wing's upper surface is moving forward and shrinking in chordwise size. At 6-deg angle of attack, the bubble has moved forward on the upper surface and transition in the bubble, followed by reattachment resulting from the transition to turbulent flow, results in an increased lift curve slope. Massive upper-surface separation results when the bubble can no longer force attached flow in the presence of large pressure gradients and advancing trailing-edge separation. At angles of attack above 16 deg, flow is separated over the entire upper surface of the wing. When the angle of attack is reduced following stall, the classic stall hysteresis loop appears in the lift curve. This occurs because, in order to regain an attached flow on the wing's upper surface, the laminar bubble must be re-established. Reformation of the laminar bubble requires a substantial decrease in angle of attack.

The "wet" wing results at Re = 100,000 are seen to be quite different from the "dry" case. Essentially, the rain has totally eliminated the normal low Reynolds number behavior of the flow. It appears that a turbulent boundary layer now exists on both the upper and lower wing surfaces. The apparent alteration of the slope of the lift curve results from elimination of laminar separation and the laminar bubble on both surfaces of the wing. Since the laminar bubble is a lift-enhancing device at higher angles of attack, its elimination results in a reduction of $C_{L_{\max}}$. Accompanying this reduction in $C_{L_{\max}}$ is, however, an



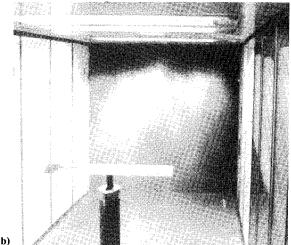


Fig. 3 a) Spray nozzle detail and b) wing behind spray in tunnel.

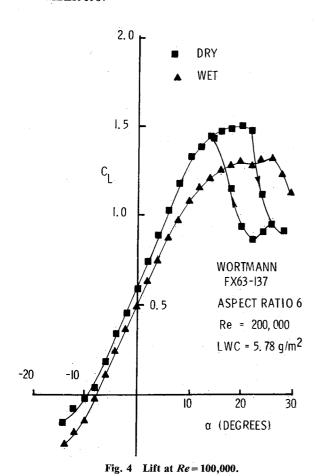
elimination of the sudden massive upper-surface separation found in the "dry" wing case and the hysteresis that accompanies that laminar bubble related separation process. The result is a more gentle stall and even a gentle secondary stall with no hysteresis.

The net result of the rain is actually an improved wing performance in terms of lift, with increased C_L at most positive angles of attack and a much more gentle stall. This is accompanied by a slight decrease in $C_{L_{\max}}$.

The drag coefficient results for these same tests at a

The drag coefficient results for these same tests at a Reynolds number of 100,000 are shown in Fig. 5. The very low drag values at this Reynolds number result in considerable scatter in the data. It appears, however, that the increase in boundary-layer turbulence due to rain has no adverse effect on drag. The scatter in these results prevents more precise conclusions regarding rain effects on drag at a Reynolds number of 100,000.

The results of similar tests at a Reynolds number of 200,000 are shown in Figs. 6 and 7. At this Reynolds number, it appears that the results of rain on lift are generally detrimental. Here the normal dry wing boundary-layer flow is more stable near the wing's leading edge than in the previous case, and the added roughness due to rain is no longer needed to promote a stable, attached flow. At higher angles of attack, the rain now appears to be promoting separation, perhaps due to water



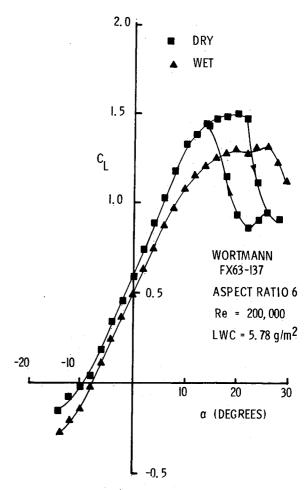


Fig. 6 Lift at Re = 200,000.

DRY

▲ WET WORTMANN

FX63-I37 ASPECT RATIO 6 Re = 200,000

LWC = 5. 78 g/m²

ا۔ 30

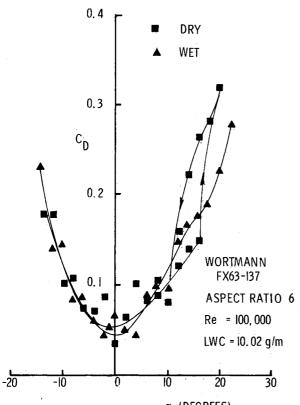
20

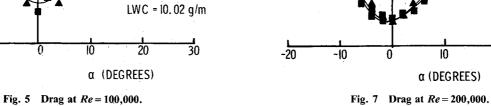
0.4

0.3

0.2

 c^{D}





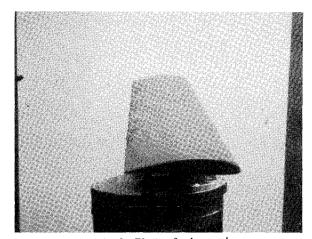


Fig. 8 Photo of rain on wing.

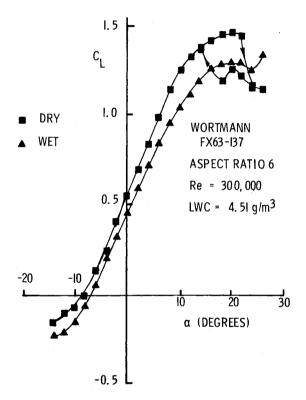


Fig. 9 Lift at Re = 300,000.

droplet accumulation and a resulting "bump" on the aft portion of the wing's upper surface where turbulent separation occurs, as can be seen in the photograph in Fig. 8. The result is a severe degradation in lift coefficient at higher angles of attack. Stall is again smoother in the "wet" case, however, this ceases to be an advantage due to the decrease in C_L at all angles of attack in this range. While the "wet" wing does not experience C_L values as low as those on the lower (stalled) portion of the "dry" wing hysteresis loop, the net result does not appear to represent any real advantage.

The drag data at Re = 200,000 again show very little difference between the wet and dry cases.

Figures 9 and 10 present the data for a Reynolds number of 300,000. The results here are very similar to the 200,000 Reynolds number case in lift. The drag results, however, indicate that at this Reynolds number, the drag is definitely increased by the rain. At this Reynolds number, the wing is beginning to behave more as it should at "normal" Reynolds numbers, where the role of the laminar bubble in determining upper-surface flow characteristics is not as great as it is at

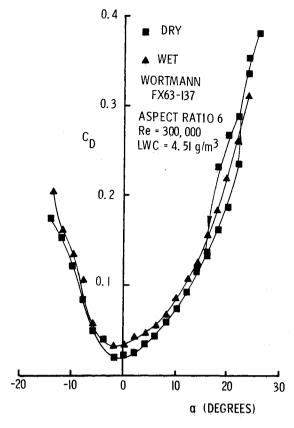


Fig. 10 Drag at Re = 300,000.

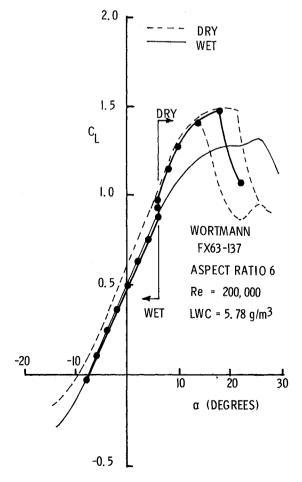


Fig. 11 C_L change with "clearing" of rain.

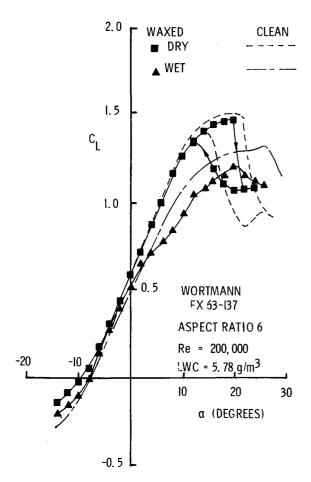


Fig. 12 Waxed wing C_L behavior.

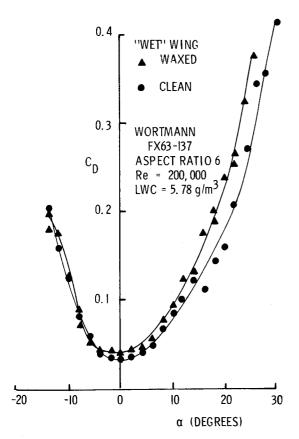


Fig. 13 Waxed wing C_D behavior.

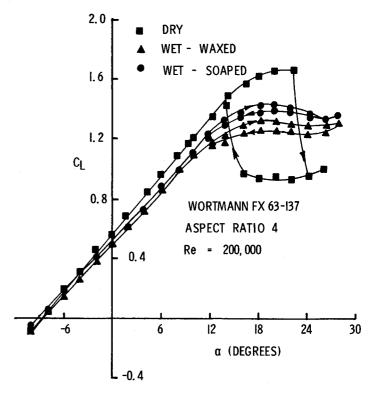


Fig. 14 Aspect-ratio-4 lift results.

lower Reynolds numbers. The presence of water droplets on the wing is definitely increasing turbulence and skin-friction drag at moderate angles of attack, having much the same effect as added surface roughness.

All other testing was done at a Reynolds number of 200,000. Here the wettability of the surface was examined as a parameter, and the ability of the wing to "clear" itself of rain effects when the rain stopped was evaluated. In Fig. 11, the C_L results are shown from a test in which the angle of attack was increased to 6 deg with rain in the flow, and then the rain was stopped to simulate the drying process that would occur if an aircraft flew out of the rain into clear air. The wing was left at an angle of attack of 6 deg, and a series of data points were taken over a period of 2 min as the wing dried. Most of the lift coefficient increase due to drying occurred within 20 s. After the 2-min pause, the angle of attack was again increased, and the curve is seen to follow the previous dry wing data plot. Stall occurred at a slightly lower angle of attack than for the dry wing, probably due to residual water droplets, which remained on the wing in the region of trailing edge turbulent separation and moved forward as the separation region advanced with increases in angle of attack.

Tests were run using a waxed wing surface to determine if the enhanced beading effect of a waxed surface would alter the effects of rain. Figure 12 compares the waxed wing wet and dry cases with the previous tests of the normal surface ("semiwettable") wing at the same Reynolds number of 200,000. It is seen that the dry cases are virtually identical, indicating that the base wing had a fairly well-polished surface and that waxing the surface did not alter its roughness.

The wet wing tests did, however, show that waxing the wing resulted in increased beading of the water and that increased roughness further reduced the lift from the unwaxed case. This is similar to the effect reported by others³ with a waxed wing surface in rain.

Figure 13 shows that waxing the wing also increased the drag over most of the range of angle of attack for the wet wing. This again indicates the increased roughness effect due to enhanced water beading on the waxed surface.

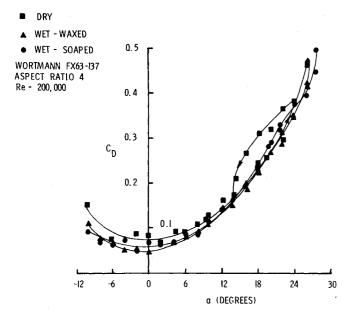


Fig. 15 Aspect-ratio-4 drag results.

An attempt was made to simulate a wettable surface, i.e., one on which there was a total sheeting of water rather than formation of droplets on the surface. This was done by coating the surface with a glycerin-based soap. This coating, however, was only temporarily successful at causing sheeting because the coating quickly washed away and the wing returned to its normal semiwettable state. Because of the inability to ascertain the true degree of wettability over any period of time during these tests, we have not presented the results here. The sheeting of the water appears to behave in a manner somewhat between the waxed and unwaxed wet wing cases tested earlier. Sheeting results in a smoother surface, which results in a decreased tendency to trip the boundary layer; however, there is still water accumulation in separated flow regions, such as the laminar bubble or at trailing-edge turbulent separation, and this tends to eliminate stall hysteresis and result in a lower $C_{L_{\max}}$, much like the $C_{L_{\max}}$ seen on the normal, unwaxed wing in rain.

A better way to investigate a fully wettable surface would be to introduce a wetting agent into the spray water itself, assuring a continued coating of the wing. This was not attempted in the present case, however, since such a case, while aerodynamically interesting, would not represent a real-life situation.

Tests were also conducted at a Reynolds number of 200,000 on the aspect-ratio-4 model. These test results are shown in Figs. 14 and 15. In this case, a complete set of tests was conducted using the soap-coating technique to evaluate the fully wettable case. The lift coefficient results (Fig. 14) are quite similar to those seen for the higher-aspect-ratio case. Figure 15 shows that the aspect-ratio-4 tests gave an apparent reduction in drag for the "wet" wing cases at most angles of attack, whereas little change was noted in the aspect-ratio-6 case. It is suspected that this may be due partly to the greater role of induced drag at the lower aspect ratio, where the lift reduction of the wet cases leads to a drag reduction.

The soaped-wing case shows that increasing the wettability of the wing surface does lead to a slight increase in $C_{L_{\rm max}}$. It is also noted that a slight hysteresis behavior appears to exist at aspect ratio 4 for the wet cases. This apparent effect needs further investigation and may be due merely to scatter in the data, although tests showed it to be repeatable behavior.

Conclusions

The somewhat unique and certainly complex nature of the aerodynamic behavior of the Wortmann FX63-137 airfoil at

low Reynolds numbers results in exposure to rain causing more profound effects than those normally seen on most airfoils. The normal influence of rain on a wing is an effective roughening of the surface, which trips the boundary layer and results in the expected effects from an increase in the extent of the turbulent boundary layer.

In the case of the Wortmann airfoil at low Reynolds numbers, the effect of rain may actually be beneficial in some circumstances. At a Reynolds number of 100,000, rain produces an increase in C_L over a wide range of angle of attack; and, while there is some reduction in $C_{L_{\max}}$, the elimination of stall hysteresis may make this a worthwhile tradeoff. While scatter in the drag data makes a firm conclusion difficult, it appears that, at this Reynolds number, rain may actually reduce C_D over much of the range of angle of attack, and there is certainly little evidence of a drag increase. For the lower-aspect-ratio test case, the drag reduction due to rain also seemed to occur at the higher Reynolds number (200,000) case.

At higher Reynolds numbers, rain seems to be largely detrimental on the aspect-ratio-6 wing, with only slight increases in drag but significant increases in lift, especially in $C_{L_{\rm max}}$. Here the elimination of stall hysteresis may make airfoil performance more predictable, but that predictability of stall behavior comes at a cost of a substantial loss in maximum lift coefficient.

A waxed surface increases water beading and causes further deterioration of wing performance. This brings into question the advisability of waxing the surfaces of low Reynolds number wings since the waxed surface seems to give no improvement in dry wing performance while making wet wing performance worse than that seen in the nonwaxed case.

It appears that a major contributor to the deterioration of performance for the wet wing case at low Reynolds number is the tendency for water to accumulate in regions of flow separation, effectively changing the shape of the airfoil. The resulting "bumps" in the surface enhance the separation process, resulting in earlier stall and lower maximum lift coefficients.

It should be pointed out that, due to a constant water flow rate in the spray system, the LWC of the flow varied inversely with Reynolds number. This is not thought to be a meaningful factor in this study since, in all cases, the LWC was sufficient to coat the wing thoroughly with water droplets. The amount of water on the wing was sufficient in all cases to cause the effects noted, and it is believed that these effects would be invariable over a much wider range of LWC values than those observed in this experiment.

References

¹Haines, P. and Luers, J., "Aerodynamic Penalties of Heavy Rain on Landing Airplanes," *Journal of Aircraft*, Vol. 20, Feb. 1983, pp. 111-119.

²Hansman, R. J. Jr. and Barsotti, M. F., "Surface Wetting Effect on a Laminar Flow Airfoil in Simulated Heavy Rain," *Journal of Aircraft*, Vol. 22, Dec. 1985, pp. 1049–1053.

³Dunham, R. E., Bezos, G. M., Gentry, C. L., and Melson, E., "Two-Dimensional Wind Tunnel Tests of a Transport-Type Airfoil in a Water Spray," AIAA Paper 85-0258, 1985.

⁴Marchman, J. F., Sumantran, V., and Schaefer, C. G., "Acoustic and Turbulence Influence on Stall Hysteresis," AIAA Paper 86-0170, Jan. 1986.

⁵Bilanin, A. J., "Scaling Laws for Testing High Lift Airfoils Under Heavy Rain," AIAA Paper 85-0260, Jan. 1985.

⁶Hastings, E. C. Jr. and Manuel, G. S., "Scale-Model Tests of Airfoils in Simulated Heavy Rain," *Journal of Aircraft*, Vol. 22, June 1985, pp. 536-540.

⁷Marchman, J. F. III and Abtahi, A. A., "Aerodynamics of an Aspect Ratio 8 Wing at Low Reynolds Numbers," *Journal of Aircraft*, Vol. 22, July 1985, pp. 628-634.

⁸Marchman, J. F. III, Abtahi, A. A., Sumantran, JV., and Sun, Z., "Effects of Aspect Ratio on Stall Hysteresis for the Wortmann Airfoil," AIAA Paper 85-1770, Aug. 1985.